ELECTROMAGNETIC FIELD GENERATED IN AIR BY A NONSTATIONARY GAMMA-RAY SOURCE IN AN IDEALLY CONDUCTING PLANE

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We consider a model problem on the generation of a radio signal by a nonstationary gamma-ray source. The problem is essentially two-dimensional in space but is reduced to a number of one-dimensional nonstationary problems. The results of a numerical solution of the problem are discussed.

A powerful gamma-ray source is capable of generating an electromagnetic field of considerable intensity. The gamma quanta propagated in the medium will give rise to a directed flux of Compton electrons. This flux plays the role of an extraneous electric current (a current generated by nonelectromagnetic forces) and at the same time causes some ionization, and hence conductivity, of the medium. The medium may be assumed to be nonionized and nonconducting in its initial state. As a result, conduction currents will arise in the medium in addition to the extraneous Compton currents.

In the spherically symmetric case these currents will obviously generate only a radial electric field. In the general case, when there are deviations from spherical symmetry (these may be due to the anisotropy of the source or to the inhomogeneity and anisotropy of the medium), the gamma rays will generate an electromagnetic field in the space surrounding the source. The problem of calculating this field is severely complicated by the need to calculate at the same time the ionization kinetics in the given real medium. Considerable difficulties arise from the fact that the process is not one-dimensional, if we consider the general case of the generation of an electromagnetic field. The nonstationarity of the gamma-ray source, which leads to nonstationarity of the entire process, is of considerable significance. The present study is devoted to the physical formulation and numerical solution of such a general problem. However, in the formulation of the problem we have made a number of simplifications and idealizations, relating to the formulation of the initial and boundary conditions, the geometry of the problem, the description of the ionization kinetics, the approximation of the nonstationary gamma-ray source, and the laws governing the propagation of the gamma quanta and the Compton electrons.

The present study contains the solution of a model problem on the generation of an electromagnetic field by a pulsed gamma-ray source situated in an ideally conducting plane surface. The medium is taken to be air of normal density at zero altitude. The electromagnetic field is assumed to be zero at the initial instant of time. It is very important that under the given physical conditions the characteristic frequencies of the electromagnetic field fall in the radio range, i.e., it is radio pulses that are generated. The problem is interesting by reason of the fact that the generation of a radio pulse comes about as the result of a chain of different but interrelated physical phenomena (gamma quanta, Compton electrons, ionization and conduction currents, electromagnetic field).

1. Kompaneets [1] and Gilinsky [2] considered the problem of the generation of a radio signal by a nonstationary gamma-ray source whose variation with angle is close to isotropic. The distribution of the

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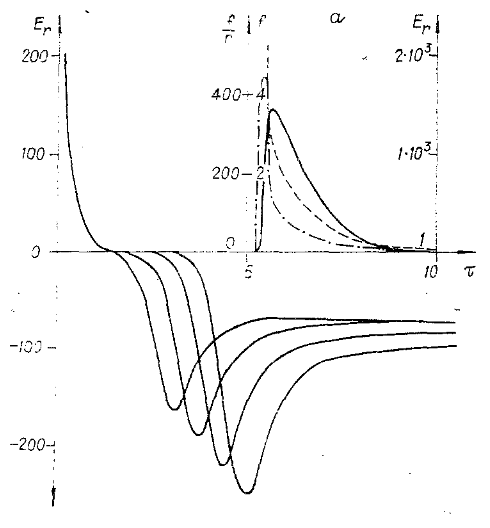

Fig. 1

Compton-electron current j and the conduction current σ have the form (we use the notation of [2])

$$j(r,\ \theta,\ t)=j_0(r,\ t)+\xi j_0(r,\ t)\cos\theta;$$

$$\sigma(r,\ \theta,\ t)=\sigma_0(r,\ t)+\xi\sigma_0(r,\ t)\cos\theta,$$

where ξ is a small parameter.

This functional relationship is not universal and is not applicable when the source is situated near or on the boundary of a conducting half-space. In [1, 2] a number of physical constants corresponding to the pressure of air at zero altitude are used. The physical formulation of the problem does not take account of the distortion of the resulting fields by the underlying conductive surface. This effect is taken into account in [3, 4], but those studies deal with a source whose intensity is not time-dependent.

In the present study we discuss a method for reducing to a number of one-dimensional nonstationary problems the essentially two-dimensional spatial problem of the electromagnetic fields generated in air by a nonstationary gamma-ray source situated directly at the boundary of a conducting half-space; the results of the solution of a model problem are given.

A numerical method for the solution of problems of this type is described in [5]. It is based on the method of characteristics. In the present study the numerical integration of Maxwell's equations was carried out by the factorization method,* which has a number of advantages: it is more convenient for calculation, since all the equations are integrated along one line, it makes it possible to take proper account of the delay in the source, and it has low sensitivity to calculation errors.

2. The physical mechanism of the transformation of a gamma-ray pulse into an electromagnetic field pulse is connected with the generation of currents of Compton electrons in air irradiated with gamma quanta from the source (as in [1, 2]).

In the model problem being analyzed, the gamma-ray source is assumed to be an isotropic point source; only the electron conductivity of the air is taken into account, and the conductivity of the lower half-space is assumed to be infinite.

The problem reduces to the integration of Maxwell's equations with currents of Compton electrons given in the form

$$j=(j(r,\ \theta,\ t),\ 0,\ 0);\quad j(r,\ \theta,\ t)=j(r,\ t)\Phi(\theta);$$

$$\Phi(\theta)=\begin{cases}1, & 0\leqslant\theta\leqslant\pi/2\\ -1, & \pi/2\leqslant\theta\leqslant\pi.\end{cases}$$

We represent this functional relationship in the form of a series of Legendre polynomials,

$$j(r,\theta,t)=j(r,t)\sum C_lP_l(\cos\theta),$$

where

$$C_l=[1-(-1)^l]\,\frac{2l+1}{4}\,\frac{\pi^{1/2}}{\Gamma\left(\frac{2-l}{2}\right)\Gamma\left(\frac{3+l}{2}\right)}\,.$$

The equations and the boundary conditions can be satisfied if we try to find a solution for the nonzero components of the electromagnetic field E_r, E_θ, H_φ in the form

$$E_r=\sum_l E_{rl}(r,t)\,P_l(\cos\theta);\qquad E_\theta=\sum_l\frac{1}{r}\,E_{\theta l}(r,t)\,P_l^1(\cos\theta);$$

$$H_\varphi=\sum_l\frac{1}{r}\,H_{\varphi l}(r,\ t)\,P_l^1(\cos\theta),$$

*The method for taking account of Maxwell's equations was proposed by A. A. Milyutin and E. I. Dinaburg.

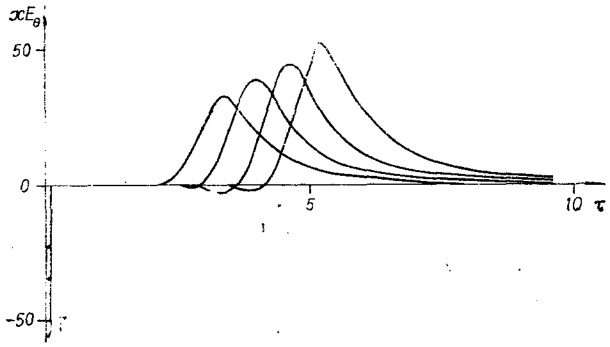

Fig. 2

where $P_l^1(x) = (1 - x^2)^{1/2} dP_l(x)/dx$ is the associated Legendre polynomial.

In this case the variation as a function of angle in the equations is considered separately, and the problem reduces to the integration of one-dimensional nonstationary equations for the coefficients of the expansion of E_{rl}, $E_{\theta l}$, $H_{\varphi l}$ for each of the l,

$$\frac{1}{c}\frac{\partial E_{rl}}{\partial t} = \frac{l(l+1)}{r^2} H_{\varphi l} - \frac{4\pi}{c}\left[\sigma(r,t)E_{rl} - j(r,t)C_l\right]; \qquad (2.1)$$

$$\frac{1}{c}\frac{\partial E_{\theta l}}{\partial t} = -\frac{\partial H_{\varphi l}}{\partial r} - \frac{4\pi}{c}\sigma(r,t)E_{\theta l}; \quad \frac{1}{c}\frac{\partial H_{\varphi l}}{\partial t} = -\frac{\partial E_{\theta l}}{\partial r} - E_{rl}.$$

The solution of the equations will be sought in the region ($a \le r \le ct$; $a/c \le t \le T$); a, T are constants. Each of the functions must satisfy a zero initial condition. The boundary conditions $E_{\theta l} = 0$ when $r = a$, $E_{\theta l} - H_{\varphi l} = 0$ and $E_{rl} = 0$ when $r = ct$ define a unique solution of the system (2.1). The first of these conditions means physically that the source is surrounded by an ideally conducting sphere of radius a, and the other two conditions must be satisfied at the front of a disturbance being propagated at the speed of light [6].

Hereafter we shall use the dimensionless coordinate $x = \mu r$ and time $y = \mu ct$, and, following [1, 2], we shall express the flux density of the Compton electrons by

$$j(r,t) = e\mu^3 l_e N \mu c e^{-x} x^{-2} f(y - x)/4\pi,$$

where $l_e \approx 3$ and $\mu^{-1} \approx 250$ m are the mean free paths of a Compton electron and a gamma quantum, respectively; e is the charge of the electron; N is the total number of gamma quanta emitted by the source; and the function f(y) describes the gamma-ray flux intensity as a function of time $\left(\int_0^\infty f(y)\,dy = 1\right)$.

In [7] the functional relationship f(t) in the stage of initial increase was expressed by the function $e^{\alpha t}$, where $\alpha = 10^8\ \text{sec}^{-1}$, and in [1, 2] and others the attenuation stage of the source was described by the function $e^{-\beta t}$, where $\beta = 10^6 \text{sec}^{-1}$.

In the present study we shall use for the function f(y) a simple interpolation process (where Ω, Δ, A are constants) which approximately describes the data given by the authors of [2, 7],

$$f(y) = \frac{1}{I}\frac{ye^{\Omega y}}{A + e^{(\Omega+\Delta)y}}; \qquad (2.2)$$

$$I = \int_0^\infty \frac{ye^{\Omega y}}{A + e^{(\Omega+\Delta)y}}\,dy.$$

The electron conductivity of the air, which is the only one taken into account, is given in the form

$$\sigma(r,t) = ek\mu^3 \nu N e^{-x} x^{-2} r(y - x)/4\pi,$$

where the dimensionless function r(y) is found from the equation

$$\frac{dr}{dy} + \frac{\gamma}{\mu c} r = \varphi(y), \quad r(0) = 0.$$

Here $k \approx 10^0$CGSE; $\gamma \approx 1.1 \cdot 10^8\ \text{sec}^{-1}$ [8] are the mobility and the sticking probability of a secondary electron, respectively; $\nu = 3 \cdot 10^4$, and the function $\varphi(y)$ describes the variation with time of the density of secondary-electron sources (the gamma-ray energy absorbed per unit of time).

The variation with time of the ionization sources at each point of the space is described by a pulse with a somewhat longer characteristic time than for the function f(y), which describes the variation of the Compton currents (because of the different contributions made by the effects of multiple quantum scattering to the values of Compton-electron current and absorbed energy). For the function $\varphi(y)$ we take an interpolation process of the same form (2.2) with the parameter $\Delta_1 < \Delta$. For such an interpolation of the current and conductivity values the ratio j/σ, defining the value of the radial polarization field, decreases

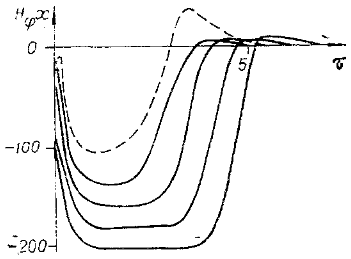

Fig. 3

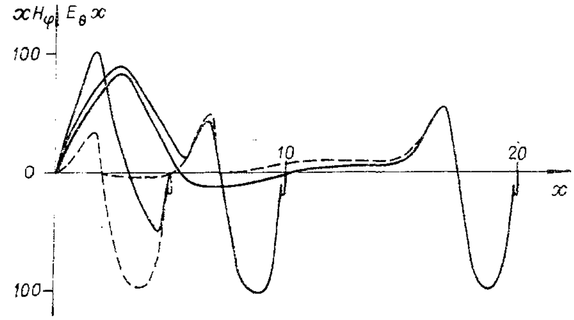

Fig. 4

with time at every point of the space when t is large. This has a substantial effect on the time-dependent evolution of the electromagnetic field that is created.

As the scale for the fields we take the quantity $E_0 = l_e \mu c/k\nu$. For the adopted values of the constants $E_0 \approx 3.6 \cdot 10^2$ V/m

$$E_{rl} = E_0 E_l;\ E_{\theta l} = \frac{E_0}{\mu}\varepsilon_l; \qquad H_{\varphi l} = \frac{E_0}{\mu} h_l.$$

Then the dimensionless functions E_l, ε_l, and h_l must satisfy the equations

$$\begin{gathered}\frac{\partial E_l}{\partial y} = \frac{l(l+1)}{x^2} h_l - \frac{4\pi}{\mu c}\left[\sigma'(x, y) E_l - \frac{1}{E_0} j'(x, y) C_l\right];\\ \frac{\partial \varepsilon_l}{\partial y} = -\frac{\partial h_l}{\partial x} - \frac{4\pi}{\mu c}\sigma'(x, y)\varepsilon_l;\\ \frac{\partial h_l}{\partial y} = -\frac{\partial \varepsilon_l}{\partial x} - E_l\end{gathered} \tag{2.3}$$

and the zero initial data and the boundary conditions

$$\varepsilon_l(x_0, y) = 0; \quad \varepsilon_l(x=y, y) = h_l(x=y, y); \quad x_0 = \mu a.$$

In Eqs. (2.3)

$$\begin{gathered}\sigma'(x, y) = \frac{4\pi}{\mu c}\sigma(x, y) = Re^{-x}x^{-2}r(y-x);\\ j'(x, y) = \frac{4\pi}{\mu c E_0} j(x, y) = Re^{-x}x^{-2}f(y-x);\\ R = ek\mu^2\nu N/c.\end{gathered}$$

In this form the problem has been completely formulated and can be integrated numerically.

3. We make the change of independent variables $x = x$, $\tau = y - x$; then the system (2.3) becomes

$$\begin{gathered}\frac{\partial E_l}{\partial \tau} = \frac{l(l+1)}{x^2} h_l - \sigma'(x, \tau) E_l + j'(x, \tau) C_l;\\ \frac{\partial \varepsilon_l}{\partial \tau} = -\frac{\partial h_l}{\partial x} + \frac{\partial h_l}{\partial \tau} - \sigma'(x, \tau)\varepsilon_l;\\ \frac{\partial h_l}{\partial \tau} = -\frac{\partial \varepsilon_l}{\partial x} + \frac{\partial \varepsilon_l}{\partial \tau} - E_l\end{gathered} \tag{3.1}$$

with the boundary conditions

$\varepsilon_l = 0$ when $x = x_0$; $\varepsilon_l - h_l = 0$, $E_l = 0$ when $\tau = 0$.

In the new plane (x, τ) the region of calculation was expanded to a rectangle ($x_0 \le x \le x_k$; $0 \le \tau \le \tau_k$, where $\tau_k = \mu c T$), and the indicated conditions became insufficient for obtaining a unique solution of the system (3.1). On the line $x = x_k$ (where x_k is a sufficiently large quantity) we specified the additional condition $\varepsilon_l = h_l$.

For the rest of the discussion it is convenient to add and subtract the second and third equations of the system (3.1) and reduce the system to the form

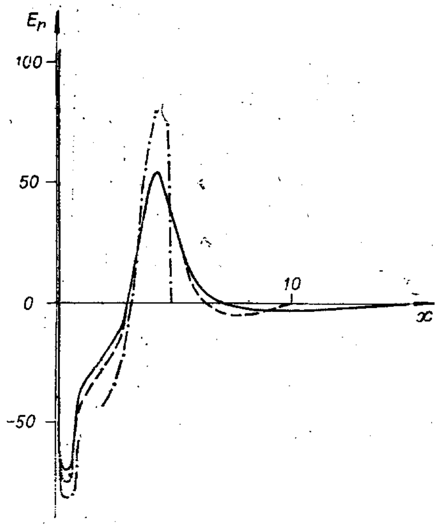

Fig. 5

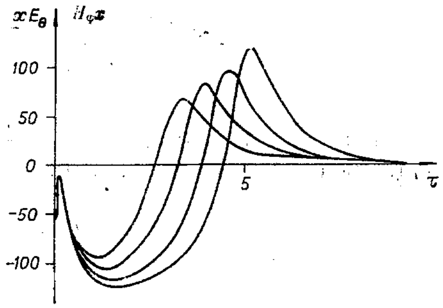

Fig. 6

$$
\begin{aligned}
\frac{\partial E_l}{\partial \tau} &= \frac{l(l+1)}{x^2} h_l - \sigma'(x,\tau) E_l + j'(x,\tau) C_l;\\
\frac{\partial(\varepsilon_l + h_l)}{\partial x} &= -\sigma'(x,\tau)\varepsilon_l - E_l;\\
2\frac{\partial(\varepsilon_l - h_l)}{\partial \tau} &= \frac{\partial(\varepsilon_l - h_l)}{\partial x} - \sigma'(x,\tau)\varepsilon_l + E_l
\end{aligned}
\tag{3.2}
$$

with the initial and boundary conditions

$$
\text{for } \tau=0\ \varepsilon_l=0,\ h_l=0,\ E_l=0;\quad \text{for } x=x_0\ \varepsilon_l=0;\quad \text{for } x=x_k\ \varepsilon_l=h_l.
$$

For a numerical solution of the problem stated above, we selected the rectangular network $\tau_n = n\Delta\tau$, $x_i = x_0 + i\Delta x$, where $n = 1, 2, \ldots N$; $i = 1, 2, \ldots I$. We made use of a symmetric approximation to the derivatives, and the desired functions at the point $i + 1/2$ were the arithmetic mean between the values of these functions at the points i and $i + 1$. Then the system of equations (3.2) can be written in finite-difference form as

$$
\begin{aligned}
&\frac{1}{\Delta\tau}\left[(E_l)^{n+1}_{i+1/2} - (E_l)^{n}_{i+1/2}\right] = (zh_l)^{n+1}_{i+1/2} - (\sigma' E_l)^{n+1}_{i+1/2} + (j')^{n+1}_{i+1/2} C_l,\\
&z = l(l+1)x^{-2};\\
&\frac{1}{\Delta x}\left[(\varepsilon_l + h_l)^{n+1}_{i+1} - (\varepsilon_l + h_l)^{n+1}_{i}\right] = -(\sigma'\varepsilon_l)^{n+1}_{i+1/2} - (E_l)^{n+1}_{i+1/2};\\
&\frac{2}{\Delta\tau}\left[(\varepsilon_l - h_l)^{n+1}_{i+1/2} - (\varepsilon_l - h_l)^{n}_{i+1/2}\right] = \frac{1}{\Delta x}\left[(\varepsilon_l - h_l)^{n+1}_{i+1} -\right.\\
&\left. - (\varepsilon_l - h_l)^{n+1}_{i}\right] - (\sigma'\varepsilon_l)^{n+1}_{i+1/2} + (E_l)^{n+1}_{i+1/2}.
\end{aligned}
\tag{3.3}
$$

The equations in ε_l and h_l obtained from the system (3.3) after eliminating the function E_l from it were solved by the factorization method, and the function E_l was calculated by using the finite-difference analog of the first equation of the system (3.2).

Investigating the selected difference scheme for stability, we can show that it is stable for any $\Delta\tau$ and Δx and gives a second-order approximation for Δx and first-order approximation for $\Delta\tau$.

The algorithm described above was applied to the special case of the problem of the generation of a radio pulse in a nonconducting medium, which admits of an analytic solution in some special cases; one of these is given in [9]. The numerical solution coincided with the analytic solution to five significant figures, which indicates the high accuracy of the above-described algorithm.

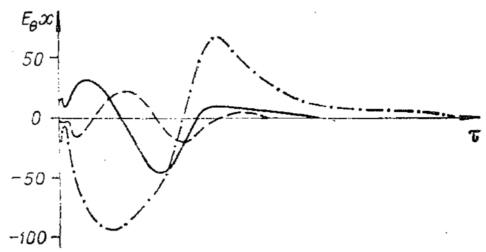

Fig. 7

4. The specific calculations were carried out for the following values of the dimensionless parameters: $\Omega = 250$; $\Delta = 8.3$; $\Delta_1 = 4$,* $A = 2.93 \cdot 10^9$; $x_0 = 0.01$; $x_k = 140$ and for values of the dimensionless coefficient R equal to $1.54 \cdot 10^5$; $1.54 \cdot 10^6$; $1.54 \cdot 10^7$; $1.54 \cdot 10^8$, corresponding to different source intensities.

The results of the calculations are shown in Figs. 1-7. On the basis of the results obtained, we can note the following features:

1. Each component of the field shows an initial short burst having a characteristic time of variation of the order of the time scale and characterizing the front of the pulse of Compton-electron currents. (The curves for various source intensities in Figs. 1-3 are for $x = 0.4$, in Fig. 6, for $x = 60$; $l = 1$.)

2. The field component E_r predominates in the initial burst at small distances, and the value of the field E_r attains its maximum value ($E_r \approx 1.76 \cdot 10^3$ for $x \approx 0.4$) when the Compton-electron current continues to increase. This is shown in Fig. 1a, where the solid curve shows the function f, the dashed curve shows f/r, and the dot-dashed curve shows the first harmonic $E_{r1}(\tau)$ for $x = 0.4$; $R = 1.54 \cdot 10^5$.

3. After the initial burst, at short distances, for some time (~ 1-3 when $x \approx 0.4$), the components of the electric field are small, while the magnetic field is large and displays a "saturation" character, particularly marked for large source intensities (Figs. 1-3).

4. After attenuation of the source for large values of t, the magnetic field disappears at short distances, and the electric field remains [Figs. 1, 4, 5 for $l = 1$; Fig. 4 shows the spatial distribution of the first harmonics $H_{\varphi 1}(x)$ (dashed curve) and $E_{\theta 1}(x)$ (solid curve) at the moments when the front of the disturbance reaches distances of 5, 10, and 20 ($R = 1.54 \cdot 10^5$); Fig. 5 shows the same for $E_{r1}(x)$].

5. As the number of the harmonic increases, the amplitude will decrease and the number of changes of sign of the field in the emitted pulse will increase (in Fig. 7, $l = 1$ for the dot-dashed curve; $l = 3$ for the solid curve; $l = 5$ for the dashed curve).

It can be seen that the initial burst of the field is caused by the time delay of the developing conductivity of the air in relation to the Compton-electron currents. From the equation for the radial polarization field, which is obtained from the first equation of the system (2.3) after the quantity $l(l+1)x^{-2}h_l$ is discarded and which describes fairly well the results of the solution of the complete system of equations at the instant of time corresponding to the initial burst, it follows that for small values of τ the conductivity is small, the air is polarized by the gamma-ray pulse, and the radial field increases. Starting at some instant of time, the conductivity limits the growth of the field at short distances and finally leads to a decrease of the radial field, although the Compton-electron current continues to increase. Because of the asymmetry of the problem it is obvious that in the zone of the currents an initial burst of a field with analogous time properties will also exist for the components E_θ and H_φ, and these properties will be propagated to greater distances, as can be seen in Figs. 4 and 6.

The most important of the properties noted above for the evolution of the fields in the zone of the source at τ values of the order of several units, at various distances, will appear at those time values for which the condition of high conductivity $2\pi\sigma t \gg 1$ is satisfied at these distances (for example, for $R = 1.54 \cdot 10^5$ at $x \approx 0.4$ this condition will be satisfied when $\tau \approx 1$-2). The boundary of the high-conductivity region moves at a rate close to the speed of light. Taking account of this and also making use of the results of [6] concerning the expansion of a sphere with finite conductivity at the speed of light in an external field, we can show, at least qualitatively, that the behavior at these instants of time of the fields found as a result of the numerical solution will coincide with the behavior of the fields produced by the gamma-ray pulse in front of a high-conductivity region boundary moving at nearly the speed of light. A quantitative estimate shows that the energy of the radial field in the initial burst is sufficient to explain, on the basis of such a model, the amplitude and duration of the emitted signals that are found by numerical integration.

Obviously, when there is a field in a nonstationary conducting medium, a space charge arises as a result of the polarization of the inhomogeneous conducting volume, and this charge does not completely

*Approximately the same relation between the parameters Δ and Δ_1 is obtained when the currents due to Compton electrons and the absorbed energy from an instantaneous source are calculated by the Monte Carlo method.

disappear as $\sigma \to 0$; therefore in the zone of the source, where there was conductivity, for large values of t there remains a static distribution of the electric field. This fact is a consequence of the original model, in which the source is attenuated exponentially with time. In another model, for example, when the intensity of a similar source decreases not to zero but to a finite value, the residual fields resulting from the nonstationary gamma-ray pulse will disappear, but in the zone of currents, for large values of t, there will appear another distribution of fields, discussed in [3].

It should be noted that the first harmonic of the Legendre-polynomial expansions of the wave field contain approximately 95% of the energy of the emitted signal, the third harmonic contains approximately 3%, etc. The fact that the number of zeros in the emitted pulse increases as the number l increases is obvious enough, since in this case the angular distribution of fields and currents in the zone of the source becomes multileaved, which, when we sum the resulting signal with the corresponding delay, leads to a larger number of changes of sign in the wave zone.

In conclusion, the authors wish to thank A. A. Milyutin and I. E. Dinaburg for working out the numerical methods used for the solution of this problem, and also thank I. N. Mikhailov and G. M. Gandel'man for their participation in the development of the problem.

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